A Bayesian Approach for Dimensionality Assessment in Psychological Networks

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Abstract

To understand psychological data, it is crucial to examine the structure and dimensions of variables. In this study, two Bayesian approaches were developed in network psychometric models to explore the dimensionality structure of the data, termed as Bayesian exploratory graph analysis or BEGA. Unlike traditional approaches that provide fixed parameter estimates, BEGA estimates posterior probabilities of graphical structures to assess the conditional dependence relations and then employs the Louvain community detection algorithm to partition and identify groups of nodes, which enables to detect the multidimensional factor structures. Monte Carlo simulations suggested that the two BEGA methods had comparable or better performance when compared with the other network-based method (EGA) and conventional parallel analysis (PA). When estimating the multidimensional factor structure, the analytically based method (i.e., BEGA.A) showed the best balance between accuracy and mean biased/absolute errors, with the second highest accuracy slightly lower than EGA but with the smallest errors. The sampling-based approach (BEGA.S) yielded higher accuracy and smaller errors than PA; lower accuracy but also lower errors than EGA. Both Bayesian techniques had more stable performance than EGA and PA across different data conditions. When estimating the unidimensional structure, the PA technique performed the best, followed closely by BEGA.A, EGA, and then BEGA.S. The study recommends using BEGA.A as an alternative tool for assessing dimensionalities and advocates for the usefulness of BEGA.S as a valuable alternate technique. The study suggested the potential to extend the regularization-based network modeling EGA method to the Bayesian framework.

Keywords: Bayesian methods, network psychometrics, dimensionality assessment, Louvain community detection algorithm, Bayesian Gaussian graphical model

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Introduction

Examining the structure and dimensions of variables is essential to understand many psychological data. Psychology typically uses measurement instruments to define a domain of functioning. Identifying the underlying dimensions of the multivariate psychological data is an important endeavor to determine how the data can be summarized into a smaller set of meaningful variables (i.e., dimensions), which are often used as inferences for psychological phenotypes. Analyzing data with overfactored dimensions often runs the risk of misspecifying the model, harming predictions, and losing knowledge translation. Furthermore, psychological theories often rely on detecting latent structures to understand human traits, in the fields such as intelligence [Garcia-Garzon et al., 2019], personality [Geiser et al., 2021], and creativity [Silvia, 2008]. Recovering the number of latent factors plays a critical role in constructing psychological theories.

Multivariate psychological data are often perceived as proxies for latent variables that interact with each other [Bollen, 2014]. Existing rules for assessing dimensions in psychology are under the latent variable modeling framework and can be classified into three categories [Garrido et al., 2016]. The first category applies: statistical tests such as maximum-likelihood, generalized least squares, and asymptotically distribution-free methods to assess dimensions. The second category relates to the mathematical and psychometric criteria including Kaiser-Guttman criterion [Kaiser, 1960], parallel analysis [Horn, 1965], and the minimum average partial method [Velicer, 1976]. The third category relies on rules of thumb, such as the scree test [Cattell, 1966] and variables with significant loadings [Floyd and Widaman, 1995] to determine the number of factors. All the above-mentioned estimation methods are based on the latent variable modeling framework, in which observed variables are believed to co-occur due to an underlying unobserved (latent) attribute that has caused the covariation between the observed variables.

Recently, a new perspective to understand psychological data has emerged with

network models. By demonstrating that a general factor model can be estimated using a fully connected network model, Van Der Maas et al. [2006] proposed using a network model to examine the dynamic relationships between variables, which rest on reciprocal mutualism or the idea that variables directly and mutually reinforce one another. Methodological advances [Borsboom et al., 2011, Borsboom and Cramer, 2013] and substantive applications [e.g., De Ron et al., 2021, Fried et al., 2015] have since increased to study psychological behaviors in this area. Epskamp et al. [2016] introduced a network model as a formal psychometric model, which proposes that symptoms, as measured by psychometric items or scale scores, are directly and reciprocally cause each other [Borsboom and Cramer, 2013, Cramer et al., 2010, van Bork et al., 2017, Van Der Maas et al., 2006]. Network models conceptualize observed variables (e.g., symptoms) as nodes, and links between nodes as edges that represent statistical relationships between symptoms or behaviors [e.g., Epskamp et al., 2016]. Although psychological network models and latent variable models possess contrasting perspectives regarding why variables are related, numerous studies have demonstrated that both models are statistically consistent under certain conditions in the binary [Chandrasekaran et al., 2010, van Bork et al., 2017], polytomous [Christensen et al., 2023], and continuous [Epskamp et al., 2016, Marsman et al., 2018, van Bork et al., 2017] data. The network structures of nodes and edges are commonly estimated in network psychometrics using Gaussian graphical model [GGM; Lauritzen, 1996] for normally distributed data. Built upon GGM, which described the conditional dependence structures of psychological constructs [Lauritzen, 1996, Wainwright et al., 2008], Golino and Epskamp [2017] developed a new assessment technique using a network modeling perspective. The technique, exploratory graph analysis [EGA; Golino and Epskamp, 2017], estimates a network and then applies a clustering algorithm to assess the factor dimensionality. Golino and Epskamp [2017], Golino et al. [2020] found an equal or superior performance of EGA to conventional latent variable based techniques in assessing factor structures. The researchers showed through a decomposition using

Woodbury matrix identity [Woodbury, 1950] that oblique factors are statistically consistent with clusters of nodes (i.e., sets of connected nodes) and orthogonal factors are statistically consistent with unconnected clusters in GGM, when the data generation mechanism is a factor model. Christensen and Golino [2021] showed that factor loadings are statistically consistent with a modified version of node strengths (i.e., sum of all connections to a node) that takes into consideration of the dimensionality structure, represented as network loadings.

Most commonly, network models are estimated using frequentist inference [Epskamp and Fried, 2018] and then assessed for dimensionality. Bayesian network estimation methods, however, possess some advantages over frequentist methods. First, rather than obtaining a fixed point estimate, the Bayesian technique provides full posterior probability distributions for the edges that capture connections between variables. The posterior distributions provide a basis for constructing the point and interval estimates, where Bayesian inferences can be easily drawn [Gelman et al., 2015]. The regularization based approach, on the other hand, studies point estimates but does not yield sampling distributions, thus posing limitations for further statistical inference [Hastie et al., 2009]. Second, by focusing on the posterior probabilities for the edges, the Bayesian approach quantifies the sampling uncertainty via probabilities. The Bayesian method numerically calculates the relational uncertainties between nodes and applies the decision rules to generate network sparsity that are consistent with substantive interpretatbility. Additionally, one's prior belief and knowledge can be incorporated into the Bayesian estimation process based on substantive theory. Thus, prior theory can play a role in determining the network structure using the Bayesian method.

This study adds a Bayesian network psychometric model that combines network science and Bayesian methodology to assess dimensionality in multivariate psychological data. The remainder of the study is organized as follows. We first review the Bayesian method for estimating the structure of the GGM and introduce a community detection

algorithm to detect patterns in the network structures. We discuss decision rules in the Bayesian estimation and develop two Bayesian network psychometric models to assess dimensionality. We investigate the performance of the proposed Bayesian techniques using two Monte Carlo simulation studies and end with discussion and future directions.

Bayesian Exploratory Graph Analysis

The study proposes a Bayesian network psychometric model, termed as Bayesian exploratory graph analysis (BEGA), to assess dimensionality in multivariate data through estimating a network structure in Gaussian graphical model [GGM; Lauritzen, 1996]. BEGA first estimates a GGM using Bayesian methods and then applies a clustering algorithm to detect the undirected weighted network clusters. This section first introduces two Bayesian approaches for estimating GGM and then discusses techniques to detect network structures and assess multidimensional and unidimensional factor structures.

GGM

GGM captures the underlying conditional dependence structure [the partial correlation network; Hojsgaard, 2008], in the multivariate normally-distributed data. Let $\boldsymbol{y}$ be a random vector of k responses and is assumed to be normally distributed, $\boldsymbol{y} \sim N_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with the mean vector $\boldsymbol{\mu} = (0_1, ..., 0_k)'$ and a $k \times k$ positive definite covariance matrix $\boldsymbol{\Sigma}$. By determining which off-diagonal elements in the precision matrix, $\boldsymbol{\Theta} = \boldsymbol{\Sigma}^{-1}$, are nonzeros, the undirected weighted network graph is obtained and used to construct an adjacency matrix. The adjacency matrix follows that $A_{ij} = \begin{cases} 1, & if\, \theta \neq 0, 1 \leq i < j < k \\ 0, & otherwise \end{cases}$. The precision matrix has a selected edge if the corresponding adjacency matrix has an element 1, and zero otherwise. The selected edges, after being standardized and reversed the sign, are partial correlations between two variables y_i and y_j, given all other variables in $\boldsymbol{y}, \boldsymbol{y}_{-(i,j)}$ on their off-diagonal elements [Epskamp et al., 2018]. The partial correlations show conditional dependency and are represented as

$$Cor(Y_i, Y_j|\boldsymbol{y}_{-(i,j)}) = -\frac{\theta_{ij}}{\sqrt{\theta_{ij}}\sqrt{\theta_{ij}}}. \quad (1)$$

A weighted network is formed using partial correlations 1, in which each variable $\boldsymbol{y}_k$ represents a node, and the partial correlations between variables are represented as edges between the nodes. A nonzero partial correlation represents the conditional dependence between nodes, whereas the zero partial correlation represents that the two nodes are independent conditional on all other nodes. Because the number of free parameters in the precision matrix can grow quadratically with the number of variables, a sparse network is typically assumed [Epskamp et al., 2017, Epskamp and Fried, 2018]. Conventionally, GGM is estimated using the penalized maximum likelihood estimation, with a variant of the *least absolute shrinkage and selection operation* [LASSO; Tibshirani, 1996] regularization technique termed *graphical LASSO* [GLASSO; Friedman et al., 2008]. On the one hand, by penalizing the model complexity while estimating the statistical model, regularization converges to the true network structure under sparse networks [Ravikumar et al., 2011]. On the other hand, the regularized partial correlation network obtains a point estimate for the edge rather than a sampling distribution, which limits statistical inference. Despite the introduction of a nonregularized bootstrapping strategy to estimate $\boldsymbol{\Theta}$ [Williams et al., 2019], Williams [2021] advised against bootstrapping LASSO penalized estimates to conduct significance tests, as the point mass at zero in sparse network likely results in the distorted sampling distribution.

Bayesian methods for GGM

This study applies Bayesian methodology to estimate GGM. GGM can be estimated either analytically or using posterior sampling in Bayesian methods [Williams, 2021]. In the analytical approach, a Wishart prior distribution, which is conjugate for the precision matrix $\boldsymbol{\Theta}$ [Kubokawa and Srivastava, 2008], is used. Following a conjugate Wishart prior $W(\nu, c\boldsymbol{I}_k)$ to $\boldsymbol{\Theta}$, with degrees of freedom ν, identity matrix $\boldsymbol{I}_k$, and a constant c, the joint

posterior density for the precision matrix $\boldsymbol{\Theta}$ follows

$$p(\boldsymbol{\Theta}|\boldsymbol{Y}) \propto p(\boldsymbol{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta}),$$

where $\boldsymbol{Y}$ is a $n \times k$ matrix drawn from a multivariate normal distribution. Due to conjugacy, the posterior distribution also has a Wishart distribution,

$$\boldsymbol{\Theta}|\boldsymbol{Y} \sim W(\nu + n, (S + c\boldsymbol{I}_k)^{-1},$$

where n is the sample size and S is the sums of squares matrix $\boldsymbol{Y}'\boldsymbol{Y}$. The posterior mode and posterior variance have closed forms, as

$$argmax_{\boldsymbol{\Theta}} p(\boldsymbol{\Theta}|\boldsymbol{Y}) = (\nu + n - k - 1)(S + c\boldsymbol{I}_k)^{-1}$$

and

$$var(\boldsymbol{\Theta}|\boldsymbol{Y}) = (\nu + n)(S + c\boldsymbol{I}_k)^{-1^2} + \boldsymbol{d}\boldsymbol{d}^{'}),$$

where ν is the degrees of freedom and $\boldsymbol{d} = diag(S + c\boldsymbol{I}_k)$. Accordingly, the analytical approach derives the graphical structure and constructs the posterior probabilities and their credible intervals for the edges.

Although the analytical approach provides an accurate and efficient estimate for the precision matrix $\boldsymbol{\Theta}$ following a Wishart distribution [Roverato, 2002], it fails to capture the sampling uncertainty as it does not produce posterior samples. A sampling-based approach can instead be used to compute the posterior distribution for the partial correlation matrix [Williams, 2021]. The sampling-based approach applies a noninformative Jeffrey's prior $|\boldsymbol{\Theta}|^{(p+1)/2}$ and derives a posterior distribution that follows a Wishart distribution $\boldsymbol{\Theta}|\boldsymbol{Y} \sim W(n-1, S^{-1})$. Sampling directly from the Wishart distribution is possible; therefore, one can draw posterior samples, $s = 1, ..., S$ to construct a posterior distribution for the $k \times k$ partial correlation matrix [Barnard et al., 2000], which can be denoted as

$$\boldsymbol{\Sigma} = -([diag(\boldsymbol{\theta})^{(s)}]^{-1}\boldsymbol{\Theta}^{(s)}[diag(\boldsymbol{\theta})^{(s)}]^{-1}),$$

where $\boldsymbol{\theta}$ are the square roots of $diag(\boldsymbol{\Theta})$. The conditional dependency and practical independent relations between edges can be determined based on the posterior probability distribution of the partial correlation matrix (Kruschke 2011).

Level of sparsity

Unlike the conventional GLASSO-based approach that applies a shrinkage on edges, Bayesian estimation does not directly produce a sparse network. Bayesian methods obtain nonzero partial correlation estimates for GGM. The nonzero small estimates typically represent weak edges, which are spurious or false positive connections in the network, even among conditionally independent nodes [Costantini et al., 2015]. In the current study, we propose penalizing model complexity by controlling for the level of sparsity.

We use Bayesian credible intervals to limit spurious edges and yield a sparse estimate for $\boldsymbol{\Theta}$ in BGGM. The Bayesian credible intervals, built upon the posterior probability distribution for $\boldsymbol{\Theta}$, summarize the posterior probability that the true edge effect is within an interval. A 90% credible interval demonstrates that there is a 90% probability that the true network structure (i.e., edges) falls within the specified interval range. While larger credible intervals indicate a higher level of sparsity in $\boldsymbol{\Theta}$, low credible intervals lead to non-zero elements in the network. In an extreme case when the credible intervals fall into zero, all the off-diagonal elements in the precision matrix are non-zeros and $\boldsymbol{\Theta}$ becomes the original Wishart posterior distribution. Specifying credible intervals is analogous to stating specificity in this context. One can relate specificity to the significance level α, mathematically denoted as $specificity = \frac{True\,Negatives}{True\,Negatives + False\,Positives} = 1 - False\,Positive\,Rate = 1 - \alpha$, and determine an interval on the directional posterior probability, which corresponds to the Bayesian p-values [Williams, 2021]. It is advisable to select a level of specificity that is consistent

with the underlying substantive theory. In other words, one establishes credible intervals based on how likely an unknown network structure lies within a particular range. The sampling-based and the analytic-based approaches converge asymptotically at a designated level of specificity [Williams, 2021]. Typically, the sampling-based approach needs a larger sample to realize the desired specificity as the technique computes the partial correlations, which can be nonnormal in small samples [van Borkulo et al., 2022].

The current study will use both the analytic-based and sampling-based Bayesian approaches to estimate GGM and compare their performances. In this study, we will adopt a 90% credible interval for the sampling-based Bayesian technique to demonstrate the confidence level of the true network structures and limit spurious edges. Researchers may designate other credible intervals to fit into their underlying theoretical framework.

Clustering algorithm

We assess the underlying factor structure of the multivariate data by applying a community detection algorithm to the estimated graphical networks. After estimating BGGM through Bayesian methods and applying the designated level of specificity to the sampling-based technique, one obtains graphical networks representing relationships between nodes. In a psychological context, the nodes having a similar psychological construct are densely connected, whereas the nodes with orthogonal constructs are expected to be further away. In previous studies that used GLASSO to estimate the GGM, the links between nodes belonging to the same construct are expected to be stronger than the connections between nodes from distinct constructs [Golino and Epskamp, 2017, Golino et al., 2020]. In this study, we use the Louvain community detection algorithm [Blondel et al., 2008] to the graphical structures to detect the optimal partitions in a network and define the dimensions in the data.

The community detection algorithms can be understood through the concept of modularity [Newman, 2006]. Modularity measures the degree of connectivity between

nodes in a community. The Louvain algorithm initiates a separate individual community for each node and records the modularity in this state; the algorithm then moves each node into a neighboring community and notes down a new modularity. The change in modularity between both states is compared. The node remains in the original community if the modularity has no gain and belongs to the adjacent community if there is a gain in modularity. This is an iterative process until the modularity does not improve and a local maxima is achieved. Hierarchical network structures can be further detected through aggregating networks in Louvain. Specifically, the algorithm moves individuals to an aggregated network, records the modularity at each state, and compares the connectivity (i.e., modularity) between states to optimize the partitions. The process is repeated until a global maxima between the expected and actual number of edges is achieved in a community [Christensen et al., 2023, Gates et al., 2016].

We suggest utilizing the Louvain algorithm for the proposed method for three primary reasons. First, Christensen et al. [2023] discovered through a well-planned simulation that when coupled with GLASSO, the Louvain algorithm was one of the most effective approaches to identify network community structures. Gates et al. [2016] had a similar finding when exploring brain network correlation structures. Second, the Louvain has the advantage of detecting hierarchical structures [Blondel et al., 2008, Jimenez, M., Abad, F. J., Garcia-Garzon, E., Golino, H., Christensen, A. P., and Garrido, 2022, Gates et al., 2016]. Bayesian methods well accommodate challenges in hierarchical data [e.g., Gelman et al., 2015, Gelman and Hill, 2002]. Proposing Louvain in BEGA allows the potential to detect hierarchical network structures in future studies. Third, compared to the Louvain community detection technique, the walktrap algorithm [Pons et al., 2006], as the default algorithm for the GLASSO-based EGA [Golino and Epskamp, 2017] method, may encounter scalability challenges as it could become computationally expensive for the walktrap technique in large or hierarchical networks [Lancichinetti et al., 2008].

Bayesian Exploratory Graph Analysis (BEGA). This study proposes a Bayesian network psychometric modeling framework for assessing factor structures, termed as Bayesian Exploratory Graph Analysis (BEGA). Specifically, we develop two BEGA models, which vary based on the way the Bayesian methods are used to determine the structure of the conditional (in)dependence. The first model estimates the conditional dependence structure using the analytical-based Bayesian approach with conjugate Wishart prior, and is termed BEGA.A. The second model, BEGA.S, adopts a sampling-based Bayesian approach to estimate a sparse network structure. In this study, we establish the interval of values in which there is a 90% probability of containing the true values, known as 90% credible intervals, in the posterior samples of BEGA.S to regulate the degree of sparseness. Researchers may opt for a value range different from 90% to account for their prior knowledge and beliefs about the network structure in practice. Both BEGA.A and BEGA.S employ the Louvain community detection algorithm to assess the undirected weighted network clusters. To address the issue of unidimensionality, we use the abovementioned expand adjustment rule [Golino et al., 2020] by creating an auxiliary dimension and adjusting unidimensionality through the auxiliary factor structure in BEGA.A and BEGA.S.

Because the true factor structure is unknown in practical settings, the BEGA techniques assess the factor structures by verifying the unidimensional structure first and then proceeding to assess the multidimensionality. Specifically, after obtaining an empirical dataset with a sample size of N, the techniques would simulate a dataset with the same sample size of N as the empirical dataset with a hypothetical factor structure consisting of four items and factor loadings of 0.7. This ensured that a non-unidimensional solution would be obtained when estimating the network structures. The process involves combining the simulated data with the actual data and using either an analytical-based or sampling-based Bayesian estimation approach to estimate the network structure. The sampling-based Bayesian estimation incorporates a designated specificity level and controls

for sparseness. The output is a network graphical structure based on the partial correlation matrix estimated using the Bayesian approaches. The Louvain community algorithm is then applied to identify community partitions. If the returned number of factors is 2 or less, the algorithm stops and records unidimensionality. If not, the process is repeated to determine multidimensionality. The final estimated multidimensional structure is calculated by subtracting one from the estimated dimension, which excludes the simulated hypothetical dimension. See Figure 1 for a detailed description of the BEGA.A algorithm; BEGA.S algorithm followed a similar procedure.

Monte Carlo Simulation Studies

We evaluated the performance of the two BEGA techniques in assessing the multidimensional and unidimensional factor structures via two Monte Carlo simulation studies. We further compared the proposed methods with two existing dimensionality assessment techniques, the GLASSO-based network psychometric tool [EGA; Golino et al., 2020] and parallel analysis using principal component analysis eigenvalues [PApca; Horn, 1965].

Simulation Study 1 - Assessing Multidimensionality

Simulation Design. In the simulation, we studied five potentially influential variables (see Table 1), including number of factors, number of items, factor loadings, interfactor correlations, and sample size. These manipulated conditions represented factor analytic scenarios commonly seen in psychological studies [e.g., Comrey and Lee, 2013, Garrido et al., 2016, Kane et al., 2005]. Specifically, the data generating factor model had one, two, three, or five factors, representing uni- or multidimensionality factor designs. where each factor had four, six, or eight items. All the models had three levels of factor loadings that represented large (0.7), medium (0.55), and low (0.4) magnitude. Because correlated factors likely affect the performance of dimensionality techniques [e.g., Garcia-Garzon et al., 2019, Garrido et al., 2016], we manipulated the interfactor

correlations to range from orthogonal (0), mild (0.3), moderate (0.5), to high (0.7). Sample sizes had three levels, representing small (250), medium (500), and large (1000) sampled conditions.

We generated data from a common factor model following a similar procedure that investigated the performance of the GLASSO-based network psychometric model, Exploratory Graph Analysis [EGA; Golino et al., 2020]. Applying the data in a similar setting allows us to compare results with established techniques including EGA and parallel analysis [Horn, 1965]. In the data generation model, first, we computed the reproduced population correlation matrix $\boldsymbol{R_R}$ as

$$\boldsymbol{R_R} = \boldsymbol{\Lambda\Phi\Lambda'},$$

where $\boldsymbol{\Lambda}$ represents the factor loading matrix and $\boldsymbol{\Phi}$ denotes the factor correlation matrix. By placing unities in the diagonal of $\boldsymbol{R_R}$ and raising the matrix to full rank, we obtained the population correlation matrix as $\boldsymbol{R_P}$. We then applied Cholesky decomposition to the population correlation matrix, such that

$$\boldsymbol{R_P} = \boldsymbol{U'U},$$

where $\boldsymbol{U}$ is the upper triangular matrix with real and positive diagonal entries. Finally, we computed the sample data matrix of continuous variables as

$$\boldsymbol{X} = \boldsymbol{ZU},$$

where $\boldsymbol{Z}$ denotes the multivariate normal distribution for the continuous variables, with rows equal to sample size and columns equal to number of items.

We applied four analytic methods to evaluate and compare the performance of the dimensionality assessment techniques in the common factor model. The first two methods were what we proposed in the study. The first method, called BEGA.A, used a Bayesian analytic approach with a conjugate Wishart prior to estimate the graphical sructure. The

method then used the Louvain community detection algorithm to extract communities. The second method, called BEGA.S, used a sampling-based Bayesian approach to obtain posterior samples. The technique set a 90% credible interval to control sparsity and used the same Louvain procedure as BEGA.A to partition clusters. The remaining two methods were established dimensionality reduction techniques for comparison purposes. Specifically, we applied the GLASSO-based EGA technique [Golino et al., 2020] as the third method. In the previous literature, Golino et al. [2020] compared EGA with other traditional dimensionality assessment techniques including Kaiser's eigenvalue-greater-than-one rule [K1; Kaiser, 1960], parallel analysis using principal component analysis eigenvalues [PApca; Horn, 1965], and parallel analysis using principal axis factoring [PApaf; Humphreys and Ilgen, 2016]. Their studies found that PApca had the overall best comparable performance to EGA over K1 and PApaf. Thus, we selected PApca as the fourth method and examined its performance in the current study to align with prior work.

All data were generated and analyses were conducted in R [Team and Others, 2013]. We ran a total of 5,000 iterations for the MCMC chains in the two Bayesian methods. A total of $3 \times 3 \times 3 \times 4 \times 3 = 324$ conditions were studied for the multidimensional design. Each simulated condition was replicated 500 times.

Evaluation Criteria. We evaluated the performance of the proposed Bayesian methods to assess the number of factors and compared them with existing dimensionality assessment techniques. We investigated the hit rate (HR), mean bias error (MBE), and mean absolute error (MAE) of the estimated number of factors across simulation replications. Let F denote the true number of factors in the population. Let $\hat{F}$ denote the estimated number of factors from the kth simulation replication. For $\hat{F}_k = F$, we counted it as hit in the kth replication; conversely, for $\hat{F}_{k+1} \neq F$, we counted it as miss in the $(k+1)th$ replication. The hit rate is defined as

$$HR = \frac{number\,of\,hit}{number\,of\,hit + number\,of\,miss}$$

, which indicates the percentage of replications correctly recovering the true number of factors from the data generation process. The hit rate ranges from 0 and 1 and can be seen as a metric for assessing accuracy.

MBE measures the average of the differences between the estimated number of factors and the true population factor. It is defined as

$$MBE = \frac{\Sigma_{k=1}^{K}(\hat{F}_k - F)}{K}.$$

MBE measures the bias of the predicted performance of the dimensionality assessment technique. A positive MBE indicates that the model is biased towards overestimating the true number of factors, while a negative MBE shows that the model underestimates the true factors. An MBE of 0 indicates no bias in the estimation.

MAE captures the average of the absolute difference between the estimated and the true number of factors. It is represented as

$$MAE = \frac{\Sigma_{k=1}^{K} \mid \hat{F}_k - F \mid}{K}.$$

MAE is a measure of the average magnitude of the errors in estimating the factor structure. While an MAE of 0 indicates no errors, higher values of MAE show greater magnitude differences in estimation errors.

To further determine the impact of the manipulated factors and their interactions on the performance of the proposed Bayesian methods, we conducted ANOVAs for each method, where the hit rate was the dependent variable and the five manipulated data conditions were the independent variables. We employed the partial eta squared statistic (η_p^2) as a metric to gauge the magnitude of the effect, with effect sizes of 0.01, 0.06, and 0.14 being categorized as small, medium, and large, respectively [Cohen, 1992].

Simulation Results. Table 3 summarized the HR, MBE, and MAE for the four models. Table 4 presented the main and interaction effects from the ANOVAs. The interaction effect with large or close to large effect sizes (i.e., $\eta_p^2 \geqslant 0.13$) was further illustrated in Figure 2.

The two Bayesian psychometric models (i.e., BEGA-A and BEGA-S) had comparable performance to EGA and PApca in terms of high hit rate (HR), low mean biased error (MBE), and low mean absolute error (MAE) across conditions examined in the study. As shown in Table 3, the BEGA.A method had the second best HRs (0.82) overall, which was lower than the EGA (0.84) method. Nevertheless, BEGA.A had the smallest MBEs and MAEs out of all four methods. At the same time, the BEGA.S method had the third-best HR (0.80), which was higher than the PApca method (0.78). The BEGA.S also had smaller and better MBEs and MAEs than both the EGA and PApca methods. The results additionally examined ANOVA interactions and effect size stabilities across manipulated conditions (see Table 4). The sub-section below broke down the performance by manipulated factors.

Factor loadings. The two Bayesian techniques performed better than EGA and PApca with higher HRs and better MBE and MAE when the factor loadings were medium to high. When the factor loading was low (0.40), EGA had the highest HR while BEGA.A had the smallest MBE and MAE. Specifically, when the factor loading was high ($\lambda = 0.7$), the two BEGA methods had the highest HRs (0.97 and 0.96, respectively), as compared to EGA (0.91) and PApca (0.88). The two BEGA methods also had the lowest MAEs (0.03 and 0.04, respectively), as compared to EGA (0.23) and PApca (0.24), as well as the lowest MBEs (-0.02 and 0.01, respectively), as compared to EGA (-0.23) and PApca (-0.24). As the factor loading decreased to 0.55, EGA and BEGA.A had the highest HRs (0.90 and 0.88, respectively), followed by BEGA.S (0.86) and PApca (0.81). At the same time, BEGA.A produced the smallest MAE (0.13), followed by BEGA.S (0.17), EGA (0.22), and PApca (0.39); following a similar pattern, BEGA.A and BEGA.S had the smallest MBEs (-0.07 and 0.07, respectively), as compared to EGA (-0.20) and PApca (-0.39). When the factor loading decreased to 0.4, the four methods performed differently in terms of HRs, MBEs, and MAEs. EGA had the highest HR, followed by PApca, BEGA.A, and BEGA.S. BEGA.A had the lowest and best MBE and MAE, followed by EGA, BEGA.S, and PApca.

Interfactor correlations. The two Bayesian techniques had apparently outstanding performance in terms of high HRs and low MBEs and MAEs under high interfactor correlation (0.7). EGA had the best performance when the interfactor correlations were 0.5, while PApca performed the best as the interfactor correlation decreased to 0.3 and lower. Specifically, when the interfactor correlations were 0.7, BEGA.A and BEGA.S demonstrated clear higher HRs (0.77 and 0.74, respectively) than EGA (0.68) and PApca (0.51), respectively. BEGA.A and BEGA.S also had better MBEs and MAEs than EGA and PApca. When the interfactor correlations were 0.50, EGA had the best HR, MBE and MAE, followed by BEGA.A. PApca had higher HR than BEGA.S while the latter had better MBE and MAE than the former. As the interfactor correlations decreased (0.3 or less), PApca had the best HR, MBE and MAE, followed by EGA, BEGA.A, and BEGA.S.

Number of factors. In the presence of five factors, BEGA.S performed the best with high HRs and low MBEs and MAEs. BEGA.A performed the best when there were 3 factors in the data, and EGA performed the best when having 2 factors in the data. Specifically, with five factors, BEGA.S had a pronounced performance with the highest HR amd the lowest MBE and MAE among the four techniques. EGA had the second best HR, while BEGA.A had the second best MBE and MAE. The PApca technique performed the least well in these three evaluation criteria under the condition. Having 3 factors in the study, BEGA.A performed the best with high HRs and low MBEs and MAE, followed by EGA, BEGA.S, and PApca, As the number of factors decreased to 2, EGA had the best performance. When compared with PApca, BEGA.A had lower HR while MBE and MAE. BEGA.S performed the least well under these conditions.

Variables per factor. With eight items per factor, while EGA had the highest HR and smallest MAE, BEGA.A had the smallest MBE among the four methods. With six items per factor, EGA had the highest HR while BEGA.A had the smallest MBE and MAE. In the presence of four items per factor, BEGA.S performed the best with the

highest HR and smallest MBE and MAE. Specifically, when each factor had eight items, EGA had the highest HR, followed by PApca, BEGA.A, and BEGA.S. BEGA.A had the smallest MBE, followed by EGA, PApca, and BEGA.S. With regards to MAE, EGA performed the best, followed by BEGA.A, PApca, and BEGA.S. When each factor had six items, EGA had better HR (0.87) than BEGA.A (0.86), while worse MBE (-0.18) than BEGA.A (-0.08) and higher MAE (0.26) than BEGA.A (0.16). As the items decreased to four per factor, the two Bayesian techniques performed the best in terms of HR, MBE, and MAE. BEGA.S had the best HR (0.83), followed by BEGA.A (0.80), EGA (0.74), PApca (0.65); BEGA.S also had the smallest MAE (0.19), followed by BEGA.A (0.26), EGA (0.52), PApca (0.65), as well as the best MBE (-0.06), followed by BEGA.A (-0.25), EGA (-0.49), and PApca (-0.71).

Sample size. When the sample sizes were medium to large (i.e., $N \geq 500$), the two BEGA methods outperformed in terms of the best HRs, MBEs and MAEs. Specifically, when $N = 1000$, the HRs for BEGA.A., BEGA.S, EGA, and PApca were 0.94, 0.95, 0.88, and 0.86, respectively; the MBEs for the four methods were -0.10, 0.13, -0.25, and -0.44, respectively; and the MAEs for BEGA.A., BEGA.S, EGA, and PApca were monotone increasing, as 0.21, 0.26, 0.31, and 0.45, respectively. A similar pattern appeared under the medium sample sized condition (i.e., N=500). BEGA.A and BEGA.S produced better or identical HRs than EGA and PApca, and smaller MBEs and MAEs than the other two techniques. As the sample size decreased to 250, EGA and PApca had higher HRs than BEGA.A, while BEGA.A had the best MBE and MAE. BEGA.S did not perform well under these conditions.

ANOVAs. We further conducted ANOVAs to assess the impact of the manipulated variables and their interactions. In the ANOVAs, the HR was the dependent variable and the five manipulated data conditions were the independent variables. The ANOVAs estimated up to four-way interactions. The effect sizes for the ANOVAs were presented in Table 4. Of note, BEGA.A was the only method that did not have an

interaction with a medium or up effect size ($\eta_p^2 \geqslant 0.06$), and BEGA.A and BEGA.S were the only techniques that did not have interactions with a large effect size ($\eta_p^2 \geqslant 0.14$). In terms of the main effects, PApca had the highest effect sizes, showing that the accuracy of PApca was largely affected by the variability of factor loadings, interfactor correlations, sample size, number of factors, and number of variables per factor.

The two-way interaction VF (variables per factor) * FC (interfactor correlations) yielded close to large effect sizes for EGA ($\eta_{p.EGA}^2 = 0.14$) and PApca ($\eta_{p.PApca}^2 = 0.13$), respectively, which demonstrated the variabilities of the EGA and PApca methods in producing accuracies. We plotted this two-way interaction which demonstrated large effect sizes (see Figure 2) to further investigate and compare the performances of the four methods. Under the four variable per factor condition, the two Bayesian techniques had relatively stable HRs across interfactor correlation conditions, whereas the performance of EGA and particularly PApca varied largely across varying interfactor correlations. Specifically, when the factors were uncorrelated, PApca had the highest HR (≈ 0.98), with BEGA.A, BEGA.S and EGA having slightly lower and similar HRs (≈ 0.92). The four methods had similar HRs (≈ 0.90) when the interfactor correlation was 0.3. As the interfactor correlations increased to 0.5, PApca had a large drop in HR to around 0.7. When the interfactor correlations climbed to 0.7, PApca had a huge plummet in HR (≈ 0.30), and so did EGA (≈ 0.40), while the two Bayesian approaches remained relatively high in accuracy ($\approx$ *between* 0.71 *and* 0.76). Similar patterns were seen when each factor had 6 items. When there were 8 items per factor, BEGA.A, BEGA.S, and EGA had stable performances across varying interfactor correlations, while PApca had sharp drop in HR when the interfactor correlation increased.

In sum, the results showed that across the studied conditions, EGA and BEGA.A had the highest HRs (0.88 and 0.87, respectively), followed by BEGA.S (0.83) and PApca (0.83); BEGA.A had the lowest and best MBE (-0.08), followed by BEGA.S (0.13), EGA (-0.18), and PApca (-0.33); and BEGA.A had the lowest and best MAE (0.16), followed by

BEGA.S (0.23), EGA (0.24), and PApca (0.34). Furthermore, across manipulated data conditions and their interactions, the BEGA.A method was the least affected by varying levels of data conditions both in the main and interaction effects, followed by BEGA.S and EGA. The performance of PApca was largely affected by varying levels of manipulated data conditions.

Simulation Study 2 - Assessing Unidimensionality

Unidimensionality remains a challenge in network psychometric models. The BEGA.A and BEGA.S models developed in the study applied the expand unidimensionality adjustment rule [Golino et al., 2020]. We carried out a second simulation study to assess the effectiveness of the expand adjustment rule proposed to the current two Bayesian models.

Data were generated using a similar design as in the multdimensional setting (see Table 1), except that the true data generating process was from a one-factor (i.e., unidimensional) model. A total of $3 \times 3 \times 4 \times 3 = 108$ conditions were studied for the unidimensional design and each condition was replicated 500 times. BEGA.A and BEGA.S started with the expand adjustment rule to verify whether there was a unidimensional factor structure. After verifying the unidimesionality, the techniques proceeded to check for multidimensionality in the absence of the unidimensionality (See Figure 1).

Table 3 presented a summary of the HR, MBE, and MAE values for the four techniques, which were defined in a similar way as in the multidimensional setting. The findings showed that BEGA.A and BEGA.S demonstrated satisfactory performance when assessing unidimensionality. In comparison to the other two existing techniques, PApca demonstrated outstanding performance, achieving almost perfect HR (99.92%). Following closely behind were BEGA.A (99.44%) and EGA (99.38%). PApca also exhibited near-zero values for MBE and MAE, with BEGA.A and EGA closely following behind. Among the four methods, BEGA.S had the lowest performance, with an HR of 92.67% and MBE and

MAE values of 0.11, although these results were still considered quite favorable.

Discussion

This study developed two Bayesian network psychmetric models to assess the dimensional structures in the data. The first model, BEGA.A computed the Gaussian graphical structures analytically from a conjugate G-Wishart prior distribution [Kubokawa and Srivastava, 2008] and used the Louvain community detection algorithm [Blondel et al., 2008] to partition the nodes and assess dimensions. The second model, BEGA.S, adopted a sampling-based Bayesian approach [Williams, 2021] while controlling for the graphical structure's sparsity and then applied the Louvain community detection algorithm [Blondel et al., 2008] to detect the dimensionality. Both approaches applied an expand adjustment rule [Golino et al., 2020] to evaluate potential unidimensional factor structure in the model. We compared the performance of the proposed two Bayesian techniques (i.e., BEGA.A and BEGA.S) with the GLASSO-based network psychometric model (i.e., EGA) and the eigenvalue-based parallel analysis (i.e., PApca) via two Monte Carlo simulation studies.

The study found that when estimating multdimensional structures, EGA had the highest HR, followed by BEGA.A, BEGA.S, and PApca. BEGA.A had the smallest and most accurate MBE and MAE, followed by BEGA.S, EGA, and PApca. Although BEGA.A approach had a slightly lower overall HR than EGA by 2.38%, it had a significantly better MBE by 32.26% and a better MAE by 60%. Given that the small difference in HR did not outweigh the large difference in MBE and MAE, BEGA.A had the best trade-off between accuracy and mean biased/absolute errors. BEGA.S had a lower overall accuracy but also lower mean biased and mean absolute errors than EGA. Further, BEGA.S had better accuracy and smaller errors than PApca. Both Bayesian techniques had more stable performance across the main and interaction effects of the data conditions than EGA and PApca. When considering the different factors studied, BEGA.A demonstrated the best balance among the four techniques. This was observed when

studying 3 or 5 factors, 4 or 6 items per factor, medium to high factor loadings (0.55 or higher), high factor correlations (0.70), and medium to large sample size (500 or more). On the other hand, EGA performed the best when dealing with 2 factors, 8 items per factor, small factor loadings (0.40), or medium factor correlations (0.50), and PApca performed the best when the factor correlations were small (0.30 or less). When the sample size was 250, EGA had the highest HR, while BEGA.A had the best MBE and MAE. In terms of estimating the unidimensional structure, the PApca technique yielded the highest HR, as well as the best MBE, and MAE. The tied EGA and BEGA.A approaches followed closely, and then BEGA.S. In sum, the study recommends using BEGA.A as an alternative tool for assessing dimensionalities and advocates the usefullness of BEGA.S as a valuable alternate technique. The BEGA modeling framework was not intended to replace existing dimensionality detection techniques such as EGA or parallel analysis but provided another view and opportunity to examine the data.

Using Bayesian methods to assess dimensionality offers certain advantages. Instead of obtaining a fixed point parameter estimate, BEGA estimates the posterior probabilities of graphical structures to evaluate the conditional dependence relationships between variables. The estimated distributional information of the structures provides additional computing information and captures uncertainty, which will ultimately facilitate constructing intervals and conducting statistical inferences. The Bayesian techniques also allow potentials to control for the sparsity level of the graphical structure, which is a unique gain that the current GLASSO-based network psychmetric models do not possess. Unlike EGA that regularizes sparsity during the regularization process in GLASSO, the Bayesian methods do not produce a sparse matrix for the graphical structures directly, but need rules to control sparsity. Establishing rules that correspond to an appropriate level of sparsity will benefit the estimation of the graphical structures. We conducted preliminary studies and found an improved accuracy in the BEGA techniques when tailoring the sparsity level to data specific conditions. We think this also explained why BEGA-S

outperformed BEGA-A in certain conditions as the former allows the potential to control sparsity of the graphical structure. Consequently, with an appropriate decision rule, BEGA.S is more likely to detect the true dimensions.

Substantively, we see BEGA methods useful for both theoretical and applied purposes. With respect to theory, the psychological literature is replete with debates regarding the factor structure of scores where replicable good fit for a multidimensional structure is difficult to find through traditional confirmatory techniques. For example, different models have been proposed for the structure of affect scores beyond a two-dimensional positive-negative structure. Although the addition of an arousal dimension (i.e., high versus low activation potential) has garnered the most empirical attention, other dimensions (e.g., responsibility/control, certainty, situational-control, depth of experience, and regulatory focus) have been offered [Baas et al., 2008, Smith and Ellsworth, 1985]. BEGA methods hold considerable promise here and in similar cases where strong factor correlations are likely in attempts that go beyond a simple, two-dimensional structure [e.g., Jorgensen et al., 2021]. In general, in literatures where there is a proliferation of theory, constructs, and measures, such as leadership [Antonakis and House, 2014], career proactivity [Jiang et al., 2023], personality [Hough et al., 2015], we see potential in BEGA methods to contribute to theoretical debates surrounding the multidimensional nature of phenomena.

We also see considerable promise for BEGA methods to be leveraged for the practical purpose of mapping a complex outcome space onto potential test batteries to optimize prediction. A good case in point is the development and validation of test batteries for predicting work performance and hiring employees. Working from a clear conceptualization and operationalization of work performance is critical to the development and weighting of component predictor test scores. It is not uncommon for a job analysis to point to 20 or more, if not dozens of, work performance components. BEGA methods could be leveraged to reduce the components to a theoretically meaningful and optimally

weighted set of criterion dimensions that guide the selection and weighting of predictor tests. In this way, BEGA methods could support traditional criterion-related validation studies as well as synthetic validation efforts [Johnson and Carter, 2010]. In the same vein as the preceding paragraph, we see potential in BEGA methods for addressing debates in the scholarly literature regarding the dimensionality of work performance [Carpenter et al., 2021, MURPHY and SHIARELLA, 1997, Rotundo, 2002].

The study had a few additional future directions to explore. First, the current study for BEGA.S predetermined a value range of 90% as the credible interval, which reflected the probability of the true value falling into the range. However, this value range was not based on prior knowledge and could potentially be updated by researchers to improve the performance of the BEGA.S method. By adjusting the probability range, researchers can change the sparseness or density of the network, which could lead to better performance of BEGA.S. Second, in the current study, noninformative priors were used in the Bayesian estimation, which may explain why the Bayesian approach did not show much improvement in small sample size conditions. The use of informative priors in future studies could potentially help to improve the performance in such conditions by incorporating previous information as additional data [e.g., Serang et al., 2014, Shi and Tong, 2017, Zhang et al., 2007]. The performance of the proposed Bayesian network psychometric framework could be enhanced by developing and incorporating informative priors.

Note that the Louvain and fast-greedy community detection algorithms reached a similar accuracy when detecting the communities from the BGGM. We proposed the Louvain algorithm for the BEGA approaches in the current study mainly due to Louvain's ability to accommodate hierarchical structures. It may not be obvious in the current simulation setup where all the data are from a single level factor structure, however, the BEGA approach with the Louvain algorithm allows potential to address the hierarchical structures such as in the bifactor model or the hierarchical factor models. We showed evidence that the expand adjustment rule performed very well in detecting the

unidimensional structures in this study. The Louvain algorithm with an adjusted hyperparameter (resolution=0.95) performs optimally and should be considered in future studies [Christensen, 2022]. Adjusting for unidimensionality remains open for future directions.

In conclusion, this study developed and systematically evaluated two approaches based on Bayesian network psychometric models, BEGA.A and BEGA.S, for the dimensionality assessment of psychological data. The results indicate that both approaches show promise as valuable alternative techniques to existing methods such as EGA and parallel analysis. Specifically, BEGA.A demonstrated the best trade-off between accuracy and estimation errors, while BEGA.S exhibited improved performance when controlling for sparsity of the graphical structure. The use of Bayesian techniques offers several advantages, such as capturing uncertainty, facilitating statistical inference, and providing opportunities to control for sparsity levels. Moreover, the Louvain algorithm employed in the BEGA approaches offers potential for addressing hierarchical structures. Future research should focus on refining aspects of the BEGA.S method, such as adjusting the credible interval value range, incorporating informative priors, and further exploring unidimensionality adjustments. By doing so, the proposed Bayesian network psychometric framework can be further enhanced, offering researchers additional tools to analyze and understand complex data structures.

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Table 1

Simulation Design

Influential Factors	# of Factor Levels	Levels
Number of factors F	4[1]	1, 2, 3, 5[1]
Number of Items per Factor I	3	4, 6, 8
Factor Loadings λ	3	0.4, 0.55, 0.7
Interfactor Correlations r	4	0, 0.3, 0.5, 0.7
Sample Size N	3	250, 500, 1000

Note:

[1]Includes the multidimensional and unidimensional designs

Table 2

HR, MBE, and MAE for BEGA.A, BEGA.S, EGA, and PApca in Multidimensionality Assessment

	Factor loading			Variables per factor			Factor correlations				Number of factors			Sample size			Total
Method	0.4	0.55	0.7	4	6	8	0	0.3	0.5	0.7	2	3	5	250	500	1000	
								Hit Rate (HR)									
PApca	0.65	0.81	0.88	0.65	0.8	0.88	0.99	0.96	0.82	0.35	0.87	0.78	0.68	0.69	0.79	0.86	0.78
EGA	0.69	0.9	0.91	0.74	0.87	0.91	0.96	0.94	0.89	0.57	0.89	0.87	0.76	0.77	0.85	0.88	0.84
BEGA.A	0.62	0.88	0.97	0.8	0.86	0.82	0.9	0.87	0.82	0.7	0.86	0.91	0.7	0.67	0.86	0.94	0.82
BEGA.S	0.58	0.86	0.96	0.83	0.83	0.74	0.87	0.84	0.79	0.69	0.79	0.82	0.79	0.59	0.85	0.95	0.8
								Mean Biased Error (MBE)									
PAPApca	-0.7	-0.39	-0.24	-0.71	-0.39	-0.23	0	-0.05	-0.3	-1.42	-0.13	-0.34	-0.86	-0.62	-0.42	-0.28	-0.44
EGA	-0.31	-0.2	-0.23	-0.49	-0.18	-0.06	-0.03	-0.04	-0.07	-0.84	-0.06	-0.15	-0.54	-0.29	-0.21	-0.24	-0.25
BEGA.A	-0.22	-0.07	-0.02	-0.25	-0.08	0.02	-0.05	-0.07	-0.1	-0.19	0.03	0.02	-0.35	-0.13	-0.11	-0.07	-0.1
BEGA.S	0.32	0.07	0.01	-0.06	0.13	0.33	0.1	0.12	0.14	0.18	0.24	0.23	-0.07	0.4	0.04	-0.04	0.13
								Mean Absolute Error (MAE)									
PAPApca	0.71	0.39	0.24	0.71	0.39	0.23	0.01	0.06	0.3	1.42	0.13	0.34	0.87	0.63	0.42	0.28	0.45
EGA	0.52	0.22	0.23	0.52	0.26	0.16	0.07	0.1	0.16	0.93	0.11	0.21	0.62	0.42	0.28	0.26	0.31
BEGA.A	0.46	0.13	0.03	0.26	0.16	0.2	0.11	0.15	0.21	0.36	0.15	0.11	0.36	0.38	0.17	0.08	0.21
BEGA.S	0.58	0.17	0.04	0.19	0.22	0.37	0.17	0.2	0.27	0.42	0.29	0.25	0.24	0.56	0.17	0.06	0.26

Note: PApca = parallel analysis with principal component analysis eigenvalues; EGA= exploratory graph analysis; BEGA.A = analytically-based Bayesian exploratory graph analysis; BEGA.S = sampling-based Bayesian exploratory graph analysis

Table 3

HR, MBE, and MAE for BEGA.A, BEGA.S, EGA, and PApca in Unidimensionality Assessment

	Factor loading			Variables per factor			Sample size			Total
Method	0.40	0.55	0.70	4	6	8	250	500	1000	
					Hit Rate (HR)					
PApca	99.76%	100%	100%	99.80%	99.96%	100%	99.78%	99.98%	100%	99.92%
EGA	98.12%	100%	100%	99.84%	99.40%	98.89%	98.46%	99.67%	100%	99.38%
BEGA.A	98.84%	99.60%	99.87%	99.98%	99.87%	98.47%	98.31%	100%	100%	99.44%
BEGA.S	85.22%	95.04%	97.76%	98.24%	96.11%	8367%	79.59%	98.42%	100%	92.67%
					Mean biased error (MBE)					
PApca	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EGA	0.02	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.00	0.01
BEGA.A	0.01	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.01
BEGA.S	0.25	0.07	0.03	0.02	0.06	0.26	0.32	0.02	0.00	0.11
					Mean absolute error (MAE)					
PApca	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EGA	0.02	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.00	0.01
BEGA.A	0.01	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.01
BEGA.S	0.25	0.07	0.03	0.02	0.06	0.26	0.32	0.02	0.00	0.11

Note: PApca = parallel analysis with principal component analysis eigenvalues; EGA= exploratory graph analysis; BEGA.A = analytically-based Bayesian exploratory graph analysis; BEGA.S = sampling-based Bayesian exploratory graph analysis

Table 4

ANOVA Effect Size for the Hit Rate Dependent Variable

Main Effect	PApca	EGA	BEGA.A	BEGA.S	Interaction Effect	PApca	EGA	BEGA.A	BEGA.S
FL	0.18	0.17	0.24	0.25	FL:VF:FC	0.08	0.02	0.01	0
VF	0.16	0.07	0.01	0.02	FL:VF:F	0	0	0.01	0.01
FC	0.6	0.28	0.07	0.06	FL:VF:N	0	0.01	0.01	0.02
F	0.12	0.06	0.1	0	FL:FC:F	0.03	0.01	0.01	0
N	0.1	0.06	0.15	0.22	FL:FC:N	0.04	0.01	0.03	0.03
Interaction Effect	PApca	EGA	BEGA.A	BEGA.S	FL:F:N	0	0	0.02	0.02
FL:VF	0	0	0.02	0	VF:FC:F	0.03	0	0	0
FL:FC	0.1	0.02	0.04	0.02	VF:FC:N	0.03	0	0	0
FL:F	0.01	0.03	0.03	0	VF:F:N	0	0	0.01	0.01
FL:N	0.01	0.09	0.04	0.08	FC:F:N	0.01	0	0	0
VF:FC	0.13	0.14	0.01	0	FL:VF:FC:F	0.02	0	0.01	0
VF:F	0	0	0.01	0.03	FL:VF:FC:N	0.03	0	0	0.01
VF:N	0	0	0.02	0.05	FL:VF:F:N	0.01	0	0.01	0.01
FC:F	0.08	0.03	0	0	FL:FC:F:N	0.02	0.01	0.02	0.01
FC:N	0.05	0.01	0	0	VF:FC:F:N	0.01	0	0	0
F:N	0.01	0	0.03	0	FL:VF:FC:F:N	0.05	0	0	0

Note: FL = factor loading; VF = variables per factor; FC = factor correlation; F = number of factors: N = sample size; PApca = parallel analysis with principal componente analysis eigenvalues; EGA = exploratory graph analysis; BEGA.A = analytically-based bayesian exploratory graph analysis; BEGA.S = sample-based bayesian exploratory graph analysis. Cell values are partial eta squated effect size estimates. The unidimensional condition was excluded from the analyses due to not crossing with the factor correlation variable.

Figure 1

Algorithm for Analytical-based BEGA (BEGA.A)

Algorithm 1 BEGA.A for Assessing Dimensionality

Require: An empirical dataset with n observations and p variables.

function CheckUnidimensionality
    Generate a unidimensional structure *simdata* with four items and factor loadings of 0.70.
    Column bind *simdata* with the empirical data *empdata* into a new data frame *expanddata*.
    Specify a conjugate Wishart prior and apply the analytical-based Bayesian method to estimate the posterior distribution of the precision matrix for *expanddata*.
    Obtain the estimated partial correlation matrix (i.e., BGGM) from the posterior mean.
    Create a network graphical structure of the estimated partial correlation matrix.
    Identify the clusters of the graphical structures using the Louvain community detection algorithm.
    Return the number of factors as the number of communities estimated by the Louvain algorithm.
    if the returned number of factors ≤ 2 **then**
        Return the number of factors $= 1$.
        Unidimensionality is detected and recorded.
    end if
end function
function CheckMultidimensionality
    if the returned number of factors > 2 **then**
        Specify a conjugate Wishart prior and apply the analytical-based Bayesian method to estimate the posterior distribution of the precision matrix for the original empirical data *empdata*.
        Obtain the estimated partial correlation matrix (i.e., BGGM) for *empdata* from the posterior mean.
        Create a network graphical structure of the estimated partial correlation matrix.
        Identify the clusters of the graphical structures using the Louvain community detection algorithm.
        Return the number of factors as the estimated number of factors in the algorithm minus 1.
        Multidimensionality is detected and recorded.
    end if
end function

Figure 2

ANOVA Interactions for BEGA.A, BEGA.S, EGA, and PApca ($\eta^2_{p.EGA} = 0.14$, $\eta^2_{p.PCA} = 0.13$)