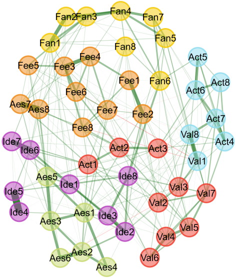
Exploratory Graph Analysis Robustness

DS-5740 Advanced Statistics



Overview: Week 11

Goals for the Week

- Learn about threats to robust measurement (and how to overcome it)
- Understand invariance and how to test it
- Cover hierarchical structures

{EGAnet} Website

Robustness of Measurement

Robustness of Measurement

validity: are you tapping into the thing you want measure?

- **Face validity** is whether your measurement appears on the surface to measure what you intend to measure
- **Convergent/discrimant validity** is whether you measurement associates with like measures as well as in the expected direction with other related (positively or negatively) measures
- Actual observable behavior is almost always preferred to self-report on those same behaviors

Robustness of Measurement

reliability: are you measurements consistent (i.e., can they be repeated)?

- **internal consistency**: whether your items are interrelated – that is, moderate ($r \geq 0.30$ to strongly correlated $r \geq 0.50$)
- **test-retest**: true "reliability" – whether your items can be repeated and are consistent each time you measure them
- Constraint: Is change in a measure due to actual change or variable responding?

Robustness of Measurement

redundancy: is there semantic similarity? is that desirable?

- More similarly items increases internal consistency but reduces breadth
- Highly similar items with some that are not similar can bias measurement
- Comes to to objective:
  - Measure one thing extremely well (higher redundancy)
  - Measure the breadth of something (minimize redundancy)

Robustness of Measurement

invariance: does the scale measure the same thing in different groups? is there demographic differences in the way the items are written

- Measurement and statistics is about generalizing – most of the time, it is desirable to ensure that our survey generalizes to different demographics
- Lack of generalizability (i.e., non-invariance) means that you *cannot* adequately compare measurement between two (or more) groups – you are no longer measuring the same thing!
- Constraints: Group differences or lack of generalizability?

Robustness of Measurement

Within the EGA framework, there are techniques that can assess these properties of measurement

- validity and redundancy: Unique Variable Analysis and Hierarchical Exploratory Graph Analysis
- reliability: Bootstrap Exploratory Graph Analysis
- invariance: Measurement Invariance

Validity

validity: whether your survey measures what it intends to measure

Network Perspective

Components represent are “causally autonomous” and mutually reinforce one another across time

Translation: components (variables) of a network should be relatively *unique* such that each one has its own causes

Key to measurement, from the network perspective, is to have unique variables in the network

Actually, for most measurement, there is a necessity to have less redundancy based on an assumption called *local independence*

This assumption states that:

> *after controlling for a latent variable (e.g., extraversion), the remaining variance of items are no longer related (i.e., zero correlations)*

Networks to not hold this assumption explicitly (there are no latent variables estimated) but the accuracy of their parameters (i.e., edge weights) is strongly affected by redundancy

Robustness of Measurement | Validity

Take a network with many variables that are fairly unique but you have the two items

1. I like to be the center of attention
2. I don't like attention

These two variables will be **strongly** connected (i.e., large edge weight)

Robustness of Measurement | Validity

Take a network with many variables that are fairly unique but you have the two items

1. I like to be the center of attention
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These two variables will be **strongly** connected (i.e., large edge weight)

When evaluating the *node strength* or the sum of the connections to each node in the network, these two variables will likely have *inflated* values

Node strength quantifies how well connected a node is in the network and many researchers take this meaning as "importance"

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When evaluating the *node strength* or the sum of the connections to each node in the network, these two variables will likely have *inflated* values

Node strength quantifies how well connected a node is in the network and many researchers take this meaning as “importance”

A question arises: Is the strength of these two nodes because they are indeed important or because they are redundant?

Unique Variable Analysis

To assess whether there is sufficient redundancy (related to multicollinearity) in a network, Unique Variable Analysis (UVA) can be applied

General Approach

1. Estimate a network (usually EBICglasso)
2. Compute weighted topological overlap (wTO) on the network
3. Apply a cut-off (≥ 0.25) to determine redundant pairs
4. Eliminate pairs based on some heuristics

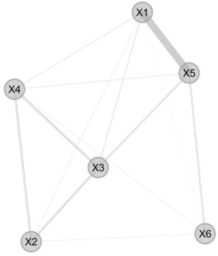
Robustness of Measurement | Validity

	X_1	X_2	X_3	X_4	X_5	X_6
P_1	3	1	4	1	3	1
P_2	5	5	5	3	5	2
...	...	...	...	...	...	...
P_i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	$x_{i,4}$	$x_{i,5}$	$x_{i,6}$

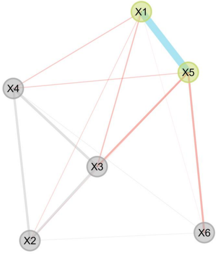
Raw Data

$$\omega_{ij} = \frac{\sum_u a_{iu}a_{uj} + a_{ij}}{\min\{k_i, k_j\} + 1 - a_{ij}}$$

Weighted Topological Overlap (wTO)



Network



Network with wTO

Robustness of Measurement | Validity

After cut-off, heuristics are used to eliminate redundant variable sets down to a single variable

2 variables: variable with the *lowest* maximum wTO to all *other* variables is retained

3 or more variables: variable with the *highest* mean wTO to all other variables in the *redundant set* is retained

Effects of Reducing Redundancy

1. More accurate dimension estimation: resolves issues associated with “minor factors” (i.e., smaller dimensions that form because of high shared variance between a smaller set of variables intend to form a dimension in a larger set)
2. More accurate edge weights: associations between variables are due less to redundancy and more to their actual contribution to the network (assuming the network captures all variables of interest)

Let's take a look at our empirical example

```
# Load {EGAnet}
library(EGAnet)

# Load data
load("../data/openness_br_dk.RData")

# Select variables of interest
openness_voi <- openness_br_dk[
  , grep("O", colnames(openness_br_dk))
]
```

Apply Unique Variable Analysis

```
# Apply UVA
openness_uva <- UVA(openness_voi)

# Print summary
summary(openness_uva)
```

```
Variable pairs with wTO > 0.30 (large-to-very large redundancy)

          node_i          node_j  wto
 O_imagination_3 O_imagination_4 0.32

----

Variable pairs with wTO > 0.25 (moderate-to-large redundancy)

                 node_i                 node_j   wto
 O_artistic_interests_1 O_artistic_interests_4 0.299
          O_intellect_2          O_intellect_4 0.299
        O_imagination_2        O_imagination_3 0.275
    O_adventurousness_2    O_adventurousness_3 0.261

----

Variable pairs with wTO > 0.20 (small-to-moderate redundancy)

                 node_i                 node_j   wto
         O_liberalism_1         O_liberalism_3 0.243
       O_emotionality_2       O_emotionality_4 0.228
 O_artistic_interests_3 O_artistic_interests_4 0.224
          O_intellect_3          O_intellect_4 0.206
        O_imagination_1        O_imagination_2 0.205
```

What are these variables though?

Using an Item Key

```
# Load codebook
openness_codebook <- read.csv("../data/openness_codebook.csv")

# Ensure proper order
openness_key <- openness_codebook$item_description[
  match(colnames(openness_voi), openness_codebook$variable_label)
]

# Apply UVA
openness_uva <- UVA(openness_voi, key = openness_key)

# Print summary
summary(openness_uva)
```

```
Variable pairs with wTO > 0.30 (large-to-very large redundancy)

           node_i                        node_j  wto
 Love to daydream. Like to get lost in thought. 0.32

----

Variable pairs with wTO > 0.25 (moderate-to-large redundancy)

                                    node_i                                             node_j   wto
        Believe in the importance of art.            Do not enjoy going to art museums. 0.299
         Avoid philosophical discussions. Am not interested in theoretical discussions. 0.299
           Enjoy wild flights of fantasy.                            Love to daydream. 0.275
 Prefer to stick with things that I know.                             Dislike changes. 0.261

----

Variable pairs with wTO > 0.20 (small-to-moderate redundancy)

                                          node_i                                             node_j   wto
 Tend to vote for liberal political candidates. Tend to vote for conservative political candidates. 0.243
                         Feel others' emotions.          Don't understand people who get emotional. 0.228
                            Do not like poetry.                  Do not enjoy going to art museums. 0.224
  Have difficulty understanding abstract ideas.       Am not interested in theoretical discussions. 0.206
                      Have a vivid imagination.                      Enjoy wild flights of fantasy. 0.205
```

By default, `UVA` will automatically remove redundancy for wTO $\geq$ 0.25 using heuristics

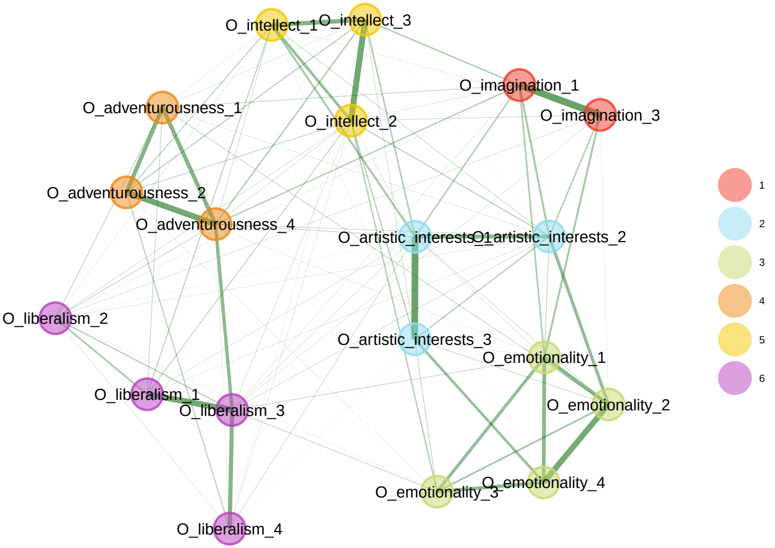
A *caveat* for this particular scale is that it was designed with fairly high redundancy for each subscale (e.g., imagination, intellect)

Still, let's proceed to EGA with the reduced data

Accessing the Reduced Data

```r
# Perform EGA
openness_reduced_ega <- EGA(openness_uva$reduced_data)
```

Robustness of Measurement | Validity



Notice anything?

Notice anything?

Our theoretical results appear *without* having to apply EGA with TEFI selection

Redundancy is a common and pervasive source of why empirical results don't match theoretical results

TEFI is not comparable to our original results because it depends on the number of variables – you must compare solutions with the same variables for TEFI to be valid

~~Reliability~~ Internal Consistency

Robustness of Measurement | Internal Consistency

Recall that reliability is whether measurement is consistent across time

Internal consistency is the extent to which items within a scale are interrelated

Much of the (psychometric) literature refers to internal consistency *as* reliability

We know better though: internal consistency $\neq$ reliability

Robustness of Measurement | Internal Consistency

In traditional psychometrics, there are many measures of internal consistency that largely depend on the type of data (i.e., continuous, polytomous, multiple dimensions, general and specific dimensions)

Internal consistency is the extent to which items within a scale are *interrelated* (i.e., correlated)

If we design a scale with proper theory and knowledge, is internal consistency meaningful?

In traditional psychometrics, there are many measures of internal consistency that largely depend on the type of data (i.e., continuous, polytomous, multiple dimensions, general and specific dimensions)

Internal consistency is the extent to which items within a scale are *interrelated* (i.e., correlated)

If we design a scale with proper theory and knowledge, is internal consistency meaningful?

Not really – we expect that our items are interrelated

In the EGA framework, a different approach is taken that aims to address a key issue and separate misunderstanding

1. Issue: internal consistency is applied scale-by-scale as if their measurement is in a silo

2. Misunderstanding: internal consistency does not mean that items are *homogeneous* or representing a single attribute

Okay… so what's the big deal?

Recall that for valid measurement we want to be sure that we are measuring some attribute

If what we are measuring in a scale isn't homogeneous, then we are capturing multiple attributes rather than a single attribute

If we are measuring multiple sub-attributes as part of a single attribute (e.g., openness to experience), then we want to be sure that each individual sub-attribute is not "contaminating" our measurement of other sub-attributes (even though they are intended to be related!)

Bootstrap Exploratory Graph Analysis

The aim of Bootstrap Exploratory Graph Analysis is to estimate the "stability" or generalizability of the EGA result

As a part of the analysis, we can quantify:

- How common our single-shot results are
- Whether our survey is *structurally consistent* – that is, whether our dimensions *remain* homogeneous and internally consistent
- Whether there are problematic items that need to be addressed

Bootstrap Approach

1. Obtain a replicate sample of data using

   parametric: random data generated from a multivariate normal distribution from the empirical sample's correlation matrix

   resampling: shuffle observations with replacement from the empirical data

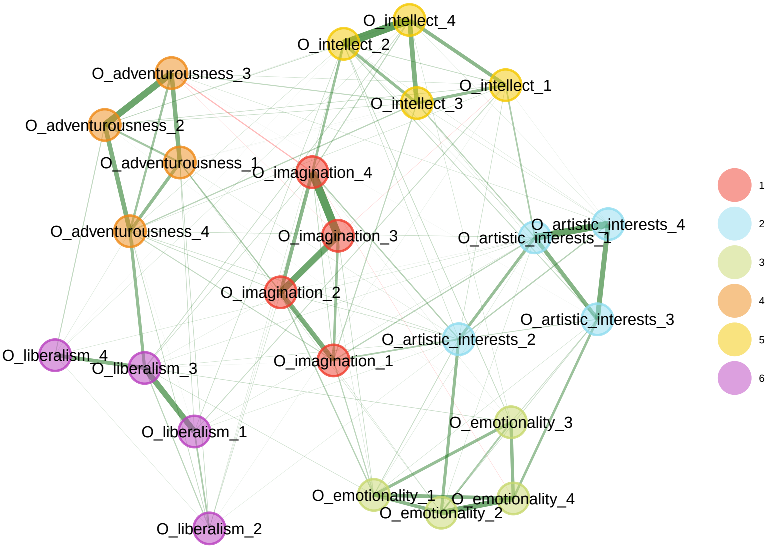
2. Apply EGA to the replicate sample
3. Repeat 1. and 2. for *N* times (e.g., 500)
4. Compute descriptive statistics (e.g., how often each number of dimensions occurs)
5. Compute item and dimension stability statistics

Apply Bootstrap Exploratory Graph Analysis

```r
# Perform Bootstrap EGA
openness_bootega <- bootEGA(openness_voi, seed = 1234)

# Summary
summary(openness_bootega)
```

Robustness of Measurement | Internal Consistency



Median Network Structure

The plot represents the *median network structure*

Across bootstraps, the median value of each edge is used to "construct" the median network structure

After, the community detection algorithm is applied

Because the algorithm is applied *after* the construction of the median network, it's not uncommon for the number of dimensions to be different from the empirical EGA

```
Model: GLASSO (EBIC)
Correlations: auto
Algorithm:  Walktrap
Unidimensional Method:  Louvain

----

EGA Type: EGA
Bootstrap Samples: 500 (Parametric)

                4     5     6
Frequency:  0.014 0.178 0.808

Median dimensions: 6 [5.14, 6.86] 95% CI
```

Item and Dimension Stability

Because items are assigned to dimensions using the community detection algorithm (unlike other dimension reduction methods such as PCA and exploratory factor analysis), additional statistics open up

item stability: how often an item, across the bootstraps, appears in the *same* dimension as the empirical EGA

structural consistency: how often an empirical EGA dimension, across the bootstraps, replicates exactly

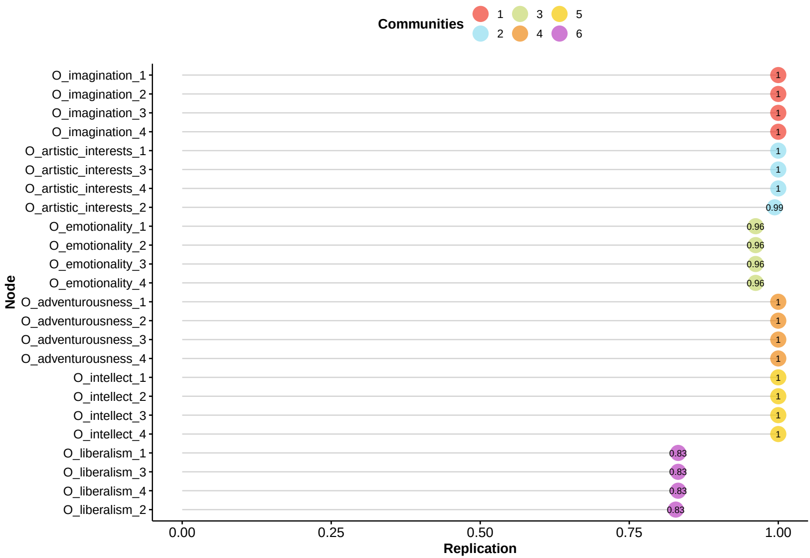
Structural consistency implicitly signals whether a dimension is internally consistent (i.e., interrelated) and explicitly signals whether a dimension is homogeneous (i.e.., remains a cohesive dimension)

Compute Item and Dimension Stability

```r
# Compute Item and Dimension Stability
openness_stability <- dimensionStability(openness_bootega)

# Summary
summary(openness_stability)
```

Robustness of Measurement | Internal Consistency



```
EGA Type: EGA
Bootstrap Samples: 500 (Parametric)

Proportion Replicated in Dimensions:

      O_imagination_1 O_artistic_interests_1       O_emotionality_1 
                1.000                  1.000                  0.962 
   O_adventurousness_1            O_intellect_1         O_liberalism_1 
                1.000                  1.000                  0.832 
      O_imagination_2 O_artistic_interests_2       O_emotionality_2 
                1.000                  0.994                  0.962 
   O_adventurousness_2            O_intellect_2         O_liberalism_2 
                1.000                  1.000                  0.828 
      O_imagination_3 O_artistic_interests_3       O_emotionality_3 
                1.000                  1.000                  0.962 
   O_adventurousness_3            O_intellect_3         O_liberalism_3 
                1.000                  1.000                  0.832 
      O_imagination_4 O_artistic_interests_4       O_emotionality_4 
                1.000                  1.000                  0.962 
   O_adventurousness_4            O_intellect_4         O_liberalism_4 
                1.000                  1.000                  0.832 

----

Structural Consistency:

    1     2     3     4     5     6 
1.000 0.994 0.962 1.000 1.000 0.828 
```

Checking for Sources of Instability

```
# Item replication in all dimensions
openness_stability$item.stability$item.stability$all.dimensions
```

Robustness of Measurement | Internal Consistency

	1	2	3	4	5	6
O_imagination_1	1	0.000	0.000	0.000	0	0.000
O_imagination_2	1	0.000	0.000	0.000	0	0.000
O_imagination_3	1	0.000	0.000	0.000	0	0.000
O_imagination_4	1	0.000	0.000	0.000	0	0.000
O_artistic_interests_1	0	1.000	0.000	0.000	0	0.000
O_artistic_interests_2	0	0.994	0.004	0.000	0	0.002
O_artistic_interests_3	0	1.000	0.000	0.000	0	0.000
O_artistic_interests_4	0	1.000	0.000	0.000	0	0.000
O_emotionality_1	0	0.038	0.962	0.000	0	0.000
O_emotionality_2	0	0.038	0.962	0.000	0	0.000
O_emotionality_3	0	0.038	0.962	0.000	0	0.000
O_emotionality_4	0	0.038	0.962	0.000	0	0.000
O_adventurousness_1	0	0.000	0.000	1.000	0	0.000
O_adventurousness_2	0	0.000	0.000	1.000	0	0.000
O_adventurousness_3	0	0.000	0.000	1.000	0	0.000
O_adventurousness_4	0	0.000	0.000	1.000	0	0.000
O_intellect_1	0	0.000	0.000	0.000	1	0.000
O_intellect_2	0	0.000	0.000	0.000	1	0.000
O_intellect_3	0	0.000	0.000	0.000	1	0.000
O_intellect_4	0	0.000	0.000	0.000	1	0.000
O_liberalism_1	0	0.000	0.000	0.168	0	0.832
O_liberalism_2	0	0.002	0.000	0.170	0	0.828
O_liberalism_3	0	0.000	0.000	0.168	0	0.832
O_liberalism_4	0	0.000	0.000	0.168	0	0.832

Some guidelines for item stability

- $\geq$ 0.70-0.75 = good stability (non-problematic variability in dimensionality)
- $\geq$ 0.40-0.50 = more than likely being split between two more more dimensions (potentially problematic multidimensionality)
- $\leq$ 0.40 = unlikely to belong in the assigned dimension

If two (or more) items are forming their own separate dimension, then it's likely that the variables are redundant (and should be picked up by UVA)

Takeaways

Although UVA suggested that there were quite a few redundancies, the overall structure of openness to experience was stable

This scale represents a caveat in the detrimental effect of redundancy – if redundancy is intended and (generally) consistent in each dimension, then measurement isn't affected

Redundancy wreaks havoc when it is unevenly spread within and between dimensions (many scales fall victim to this type of redundancy)

Sidebar

Remember EGA with TEFI? You can Bootstrap EGA and get item/dimension stability with that too:

```
# Perform Bootstrap EGA with TEFI
openness_bootega_fit <- bootEGA(
  openness_voi, EGA.type = "EGA.fit", seed = 1234
)

# Summary
summary(openness_bootega_fit)

# Compute dimension stability statistics
openness_stability_fit <- dimensionStability(
  openness_bootega_fit
)

# Summary
summary(openness_stability_fit)
```

Invariance

So far, we've performed EGA on our sample which consists of data from two countries: Brazil and Denmark

One question we might have would be whether our openness to experience survey is measured the same in the two countries

Other differences that are of common interest:

- age: "Grandpa, do you like roller coasters?"
- race/ethnicity
- interventions
- A-B testing

With measurement invariance, our goal is to determine if there is a statistical difference in our measurement of one sample versus one (or more) other samples

In traditional psychometrics, there is a methodical procedure to check levels of invariance:

- **configural**: whether dimensions are the same
- **metric**: whether loadings are the same
- **intercepts/means**: whether intercepts and means are the same

For our purposes, we'll focus on metric invariance at the item-level, which will tell us what specific items are not being measured the same

Before getting into the procedure for metric invariance, we need to talk about network loadings

loadings: relative weights for each item of how well they measure each dimension

Network Loadings

Node strength (S) for node i

$$S_i = \sum_{j=1}^{n} |\mathbf{W}_{ij}|$$

where **W** is the network

Network Loadings

Node strength split by each community (c) for node i

$$\ell_i = \sum_{j=c}^{C} |w_{ij}|$$

where C is the total number of communities and $j = c$ represents node j in community c

Network Loadings

Standardized node strength by each community *c* for node *i*

$$\aleph_{ic} = \frac{\ell_{ic}}{\sqrt{\sum \ell_c}}$$

Network Loadings

Effect sizes

- small = 0.15
- moderate = 0.25
- large = 0.35

Variables should have *at least* a small effect size on their assigned dimension

Interpretation

> *each node's contribution to the emergence of a coherent dimension in the network*

Compute Network Loadings

```
# Compute network loadings
openness_loadings <- net.loads(openness_bootega$EGA)
# We use the `$EGA` output, which is the same output
# as `EGA()`

# Summary
summary(openness_loadings)
```

```
Loading Method: BRM

                       1     2     3     4     5     6
O_imagination_3         0.45
O_imagination_2        0.393
O_imagination_4        0.331
O_imagination_1        0.215
O_artistic_interests_1       0.382
O_artistic_interests_4       0.364
O_artistic_interests_3       0.274 0.133
O_artistic_interests_2       0.139 0.114
O_emotionality_4                   0.374
O_emotionality_2             0.119 0.324
O_emotionality_1                   0.303
O_emotionality_3                   0.241
O_adventurousness_3                      0.369
O_adventurousness_2                      0.369
O_adventurousness_4                      0.279       0.152
O_adventurousness_1                      0.273
O_intellect_4                                  0.448
O_intellect_2                                  0.326
O_intellect_3                                   0.29
O_intellect_1                                  0.203
O_liberalism_3                                       0.381
O_liberalism_1                                        0.26
O_liberalism_4                                       0.148
O_liberalism_2                                       0.109
Standardized loadings >= |0.10| are displayed. To change this 'minimum',
use `print(net.loads_object, minimum = 0.10)`
```

Invariance

With invariance, the question we ask is: Are there nodes that differentially contribute to the emergence of dimension? That is, does the dimension's meaning differ?

To investigate invariance, either a theoretical or EGA-derived structure can be used

A permutation approach is used to provide a *non-parametric* test of differences

Invariance

The permutation approach works as follows:

1. Compute the empirical network loadings for each group based on some structure
2. Obtain the difference between the loadings on the *assigned* community (original difference)
3. Repeat *N* (e.g., 499) times: shuffle observations between groups, compute network loadings, and loading differences (permutated differences)
4. Compute *p*-value for *each* node based on the absolute original difference} that is greater than or equal to the absolute permutated differences

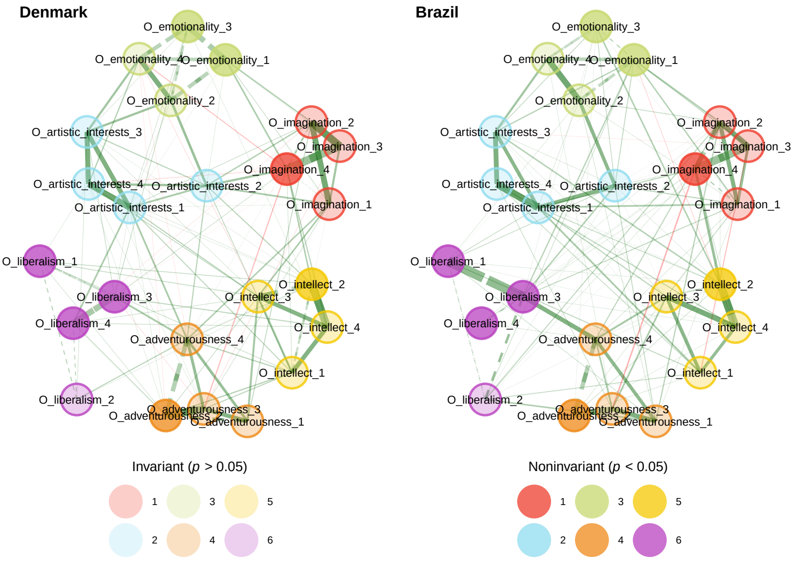
Estimate Metric Invariance

```r
# Set up theoretical dimensions
theoretical <- rep(1:6, times = 4)

# Estimate metric invariance
openness_invariance <- invariance(
  data = openness_voi,
  groups = openness_br_dk$country,
  structure = theoretical,
  seed = 1234 # don't forget the seed
)

# Plot
plot(openness_invariance)

# Summary
summary(openness_invariance)
```


Denmark
Brazil
O_emotionality_3
O_emotionality_4
O_emotionality_1
O_emotionality_2
O_artistic_interests_3
O_artistic_interests_4
O_artistic_interests_2
O_artistic_interests_1
O_imagination_2
O_imagination_3
O_imagination_4
O_imagination_1
O_liberalism_1
O_liberalism_3
O_liberalism_4
O_liberalism_2
O_intellect_2
O_intellect_3
O_intellect_4
O_intellect_1
O_adventurousness_4
O_adventurousness_3
O_adventurousness_2
O_adventurousness_1
Invariant (p > 0.05)
Noninvariant (p < 0.05)
1
2
3
4
5
6

Robustness of Measurement | Invariance

	Membership	Difference	p	p_BH	sig	Direction
O_imagination_1	1	0.011	0.744	0.893		
O_imagination_2	1	0.028	0.450	0.755		
O_imagination_3	1	0.054	0.114	0.274		
O_imagination_4	1	-0.087	0.022	0.066	*	Denmark < Brazil
O_artistic_interests_1	2	-0.082	0.062	0.165	.	
O_artistic_interests_2	2	0.012	0.726	0.893		
O_artistic_interests_3	2	0.041	0.270	0.498		
O_artistic_interests_4	2	0.011	0.788	0.901		
O_emotionality_1	3	0.127	0.002	0.012	**	Denmark > Brazil
O_emotionality_2	3	0.017	0.708	0.893		
O_emotionality_3	3	0.204	0.002	0.012	**	Denmark > Brazil
O_emotionality_4	3	-0.019	0.628	0.887		
O_adventurousness_1	4	0.000	0.996	0.996		
O_adventurousness_2	4	0.088	0.020	0.066	*	Denmark > Brazil
O_adventurousness_3	4	0.050	0.188	0.376		
O_adventurousness_4	4	0.002	0.946	0.987		
O_intellect_1	5	0.046	0.154	0.336		
O_intellect_2	5	0.106	0.008	0.038	**	Denmark > Brazil
O_intellect_3	5	-0.008	0.834	0.910		
O_intellect_4	5	-0.022	0.548	0.822		
O_liberalism_1	6	-0.232	0.002	0.012	**	Denmark < Brazil
O_liberalism_2	6	-0.035	0.472	0.755		
O_liberalism_3	6	-0.135	0.014	0.056	*	Denmark < Brazil
O_liberalism_4	6	0.223	0.002	0.012	**	Denmark > Brazil

Signif. code: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 'n.s.' 1

Brazil > Denmark

O_imagination_4: “Like to get lost in thought”

O_liberalism_1: “Tend to vote for liberal political candidates”

O_liberalism_3: “Tend to vote for conservative political candidates” (reverse)

These tend to be indicators of people who are *more* open to experience in Brazil relative to Denmark

Denmark > Brazil

O_emotionality_1: "Experience my emotions intensely"

O_emotionality_3: "Rarely notice my emotional reactions" (reversed)

O_adventurousness_2: "Prefer to stick with things that I know" (reversed)

O_intellect_2: "Avoid philosophical discussions" (reversed)

O_liberalism_4: "Believe that we should be tough on crime" (reversed)

These tend to be indicators of people who are *more* open to experience in Denmark relative to Brazil

Takeaways

- **Cannot** conclude that our measurement of openness to experience is invariant between Brazil and Denmark

- What makes a person open to experience in these countries, relative to one another, differs

- These differneces *could* have important meaning such as differences in culture and politics

Higher-order

Structures in nature often have some hierarchical organization

- cell $\rightarrow$ tissue $\rightarrow$ organ $\rightarrow$ organ system $\rightarrow$ organism
- neuron $\rightarrow$ neuronal ensemble $\rightarrow$ region (medial prefrontal cortex) $\rightarrow$ lobe (frontal) $\rightarrow$ brain

Many psychological phenomena are theorized to have a similar hierarchical structure (e.g., personality)

- nuance (imagination item) $\rightarrow$ facet (imagination) $\rightarrow$ trait (openness to experience) $\rightarrow$ meta-trait (plasticity)

Hierarchical EGA

These hierarchies can be extracted using EGA by leveraging features of the Louvain algorithm

Recall

- For each node, identify the community that *maximizes* the gain in modularity
- If there is a gain, then add that node to the community; otherwise, leave in current community
- Repeat for each node
- "Merge" nodes by summing the connections between nodes in their respective communities
- Repeat process until modularity cannot be increased *or* structure is unidimensional (all one community)

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Hierarchical EGA

The Louvain algorithm is itself "multi-level" or hierarchical

However, the merging process does not take advantage of the fact that we have additional information – that is, our data

An alternative "merging" process would be to leverage network loadings, which can then be multiplied by the data (effectively creating "network scores")

With these network scores, we can re-apply the Louvain (or some other) algorithm

Hierarchical EGA

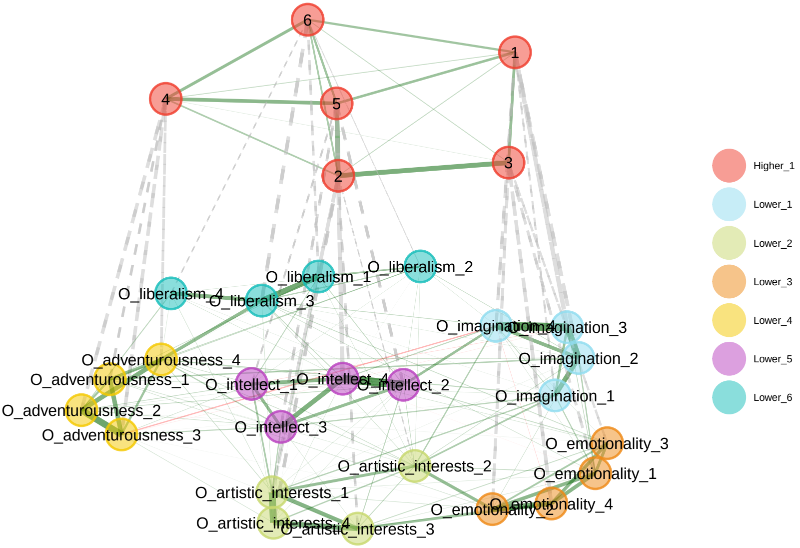
1. Estimate network and apply “lower-order” Louvain (i.e., the first pass of the Louvain algorithm)
2. Compute network loadings on these communities
3. Multiply loadings by data to get network scores
4. Estimate network on network scores and apply a community detection algorithm

Estimate Hierarchical EGA

```r
# Estimate hierarchical structure
openness_hierarchy <- hierEGA(openness_voi)

# Summary
summary(openness_hierarchy)
```

Robustness of Measurement | Higher-order



```
Lower Order

Consensus Method: Most Common (1000 iterations)
Algorithm: Louvain
Order: Lower

Number of communities:  6

      O_imagination_1 O_artistic_interests_1       O_emotionality_1    O_adventurousness_1
                    1                      2                      3                      4
        O_intellect_1         O_liberalism_1       O_imagination_2 O_artistic_interests_2
                    5                      6                      1                      2
     O_emotionality_2    O_adventurousness_2         O_intellect_2         O_liberalism_2
                    3                      4                      5                      6
      O_imagination_3 O_artistic_interests_3       O_emotionality_3    O_adventurousness_3
                    1                      2                      3                      4
        O_intellect_3         O_liberalism_3       O_imagination_4 O_artistic_interests_4
                    5                      6                      1                      2
     O_emotionality_4    O_adventurousness_4         O_intellect_4         O_liberalism_4
                    3                      4                      5                      6

----

TEFI: -22.739
```

```
Higher Order

Algorithm:  Louvain

Number of communities:  1

1 2 3 4 5 6
1 1 1 1 1 1

----

Unidimensional Method: Louvain
Unidimensional: Yes

----

TEFI: -19.924

----

Generalized TEFI: -42.663
```

Preliminary Work

Lower-order TEFI $<$ higher-order TEFI: fit in favor of correlated dimension structure over hierarchical structure

Lower-order TEFI $>$ higher-order TEFI: fit in favor of correlated dimension structure over hierarchical structure

Our result: lower-order TEFI (-22.739) $<$ higher-order TEFI (-19.924) suggesting that there is *not* evidence for the hierarchical structure

Structural Consistency

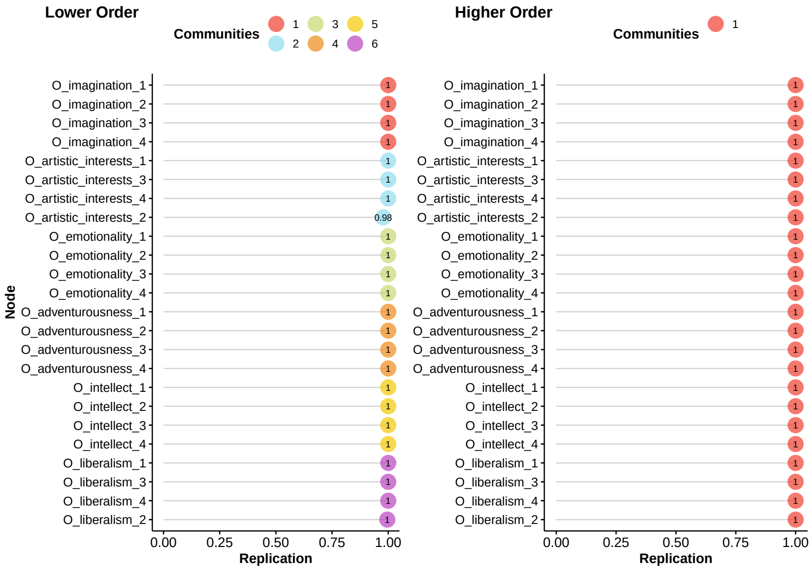
```r
# Perform Bootstrap EGA with TEFI
openness_bootega_fit <- bootEGA(
  openness_voi, EGA.type = "hierEGA", seed = 1234
)

# Summary
summary(openness_bootega_fit)

# Compute dimension stability statistics
openness_stability_fit <- dimensionStability(
  openness_bootega_fit
)

# Summary
summary(openness_stability_fit)
```

Robustness of Measurement | Higher-order



Robustness of Measurement

Wrap-up

- validity and redundancy: Unique Variable Analysis and Hierarchical Exploratory Graph Analysis
- reliability: Bootstrap Exploratory Graph Analysis
- invariance: Measurement Invariance